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(Residential Autonomous College affiliated to University of Calcutta)
B.A./B.Sc. SECOND SEMESTER EXAMINATION, JUNE 2022
FIRST YEAR (BATCH 2021-24)
Date   : 22/06/2022
MATHEMATICS (HONOURS)
Time   : 11.00 am – 1.00 pm   **Paper : IV [CC4]**   Full Marks : 50

Group – A

Answer <u>all</u> the questions. Maximum you can score is 30.

1. Find a row reduced echelon matrix which is row-equivalent to

$$A = \begin{bmatrix} 1 & -i \\ 2 & 2 \\ i & 1+i \end{bmatrix}$$

   Hence find all the solutions of $AX = 0$.   (2+2)

2. Give an example of a matrix which is in row reduced form but not in row reduced echelon form.

   Write down all 2×2 row reduced echelon matrices explicitly.   (2+2)

3. Test the following statement by citing an example:

   For a matrix *A,* rank (*A*) + nullity (*A*) = Number of columns in *A*.   (4)

4. Let *V* be the set of all pairs (x, y) of real numbers, and let *F* be the field of real numbers.

   Define

$$(x, y) + (x_1, y_1) = (x + x_1, y + y_1)$$
$$c(x, y) = (cx, y).$$

   Is V, with these operations, a vector space over the field of real numbers?   (4)

5. Define linear independence of a set of vectors.

   Find three vectors in $\mathbb{R}^3$, which are linearly dependent but any two of them are linearly independent.   (4)

6. Let *T* be the linear operator on $\mathbb{R}^3$ defined by

$$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3).$$

   What is the matrix of *T* with respect to the ordered basis $\alpha_1, \alpha_2, \alpha_3$, where

   $\alpha_1 = (1,0,1), \alpha_2 = (-1,2,1), \alpha_3 = (2,1,1).$ (4)

7. Let W_1 and W_2 be two subspaces of a vector space *V* such that their set theoretic union is also a

   subspace. Prove that one of the subspaces $W_i (i = 1, 2)$ must be contained in another.   (4)

8. Find out the rank of the following matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

                    (4)

9. Suppose U is a subspace of V and $v, w \in V$. Then show that the following statements are equivalent.

a) $v - w \in U$;

b) $v + U = w + U$;

c) $(v + U) \cap (w + U) \neq \phi$ (4)

Group – B

*Answer **all** the questions. Maximum you can score is 20.*

10. Show that for the curve $x = a + b \log\left[b + \sqrt{b^2 - y^2}\right] - \sqrt{b^2 - y^2}$ sum of the sub-tangent and sub-normal is constant. (4)

11. Find pedal equation of the curve $c^2 \left(x^2 + y^2\right) = x^2 y^2$. (4)

12. Find all the rectilinear asymptotes of the curve $y = a \log \sec\left(\dfrac{x}{a}\right)$. (4)

13. Find the envelope of the family of circles drawn on the radii vectors of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ as diameter. (4)

14. Find the radius of curvature of $y = xe^{-x}$ at its maximum point. (4)

15. Show that every point in which the curve $y = c \sin\left(\dfrac{x}{a}\right)$ meets the axis of x is a point of inflexion. (4)

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